SYMGraph: A Neural Symbolic Approach with Numerically-Aware Graph for Discrete Reasoning

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Abstract

Numerical reasoning is essential for reading comprehension, which requires symbolic numerical understanding and complex reasoning. Most existing approaches rely on specialized neural modules, which are hard to adapt to various domains or be applicable to multi step reasoning. In this work, we aim to develop a hybrid neural-symbolic approach for performing numerical reasoning (specifically on the DROP dataset). We present SYMGraph, which includes a BERT encoder, a graph neural network(GNN) and a symbolic function module. Our main point is to handle the last restriction of NumNet, having output layers restricted to question type. We address this by outputting a program which has operators defined both for answering numerical questions as well as in-text span based questions, which involve simple arithmetic as well as extraction. Moreover, our model generates a program and executes it directly, instead of using expression templates. DROP is a machine reading comprehension dataset that tests the comprehensive understanding of paragraphs and includes arithmetic, counting, and sorting, as well as a strong contextual understanding of the text, and we show a 34.99%/35.95% absolute improvement of EM/F1 by SYMGraph over NABERT.

Introduction

Machine reading comprehension (MRC) is promptly becoming a proxy for gauging a model's capabilities of natural language understanding and reasoning. One of the most commonly used reading comprehension datasets in recent years is SQuAD (Rajpurkar et al. 2016). In this dataset, the answers to the questions are spans of text contained within the passages. Numerous attention-based models have been developed that have achieved high performance nearing or exceeding human performance, such as QANet (Yu et al. 2018) and BiDAF (Seo et al. 2016), as well as many models taking advantage of BERT (Devlin et al. 2018) word embeddings.

These attention-based models face huge challenges when questions become more complex since it requires multiple steps of reasoning against paragraphs, especially when they involve discrete and symbolic operations. More specifically, we are focusing on a large-scale MRC data set named DROP(Dua et al. 2019), where the model needs to comprehend the complex structure and semantic relation of the questions firstly, retrieve corresponding information from the given passage, and then achieve question-answering with various reasoning mechanism or manipulating textual information. DROP presents both opportunities and challenges to researchers and requires a comprehensive reasoning as well as a jointly understanding of both the given passages and questions.

The existing methods that try to answer these compositional questions can be sorted into two broad categories: semantic parsing and pre-trained language model. Semantic parsing, such as SRL, maps natural language question to a expression and then yield the answers; however, it is limited to answering questions against structured knowledge and has a strong restriction of question type. Recently, some pre-trained language models, lile BERT for solving the DROP task, must follow some specialized output modules corresponding to each type of question. These specialized output modules aim at achieving different numerical reasoning, especially four types: (1) span; (2) arithmetic expression; (3) count number; (4) negation on numbers in DROP dataset. Evidently, the specialized output modules rely on the handcrafted decoder, which needs human to predefine the set of answer types. However, it is difficult to scale this model to multi-step complex reasoning. Our main point is to handle the restriction of these handcrafted decoder, having output layers restricted to question type.

In this work, we propose the SYMGraph for discrete reasoning over paragraphs, which consists of three parts: (a) an encoder(BERT) that encodes text into a vector; (b) a graph neural network to explicitly represent the numerical relationships; and (c) a symbolic module that includes a function generator and function calculator. It can scale better than other approaches to multi-step complex reasoning because it does not differ the question type and feed the whole question-answer pairs into the same following modules. Compared with the newest model NumNet(Ran et al. 2019), our model outputs a program after the graph neural network, rather than having several different output layers

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dependent on the question type. In other words, the main point of this work is to handle that last restriction of NumNet(Ran et al. 2019), having output layers restricted to question type. Our work paper addresses this by outputting a program which has operators defined both for answering numerical questions as well as in-text span based questions.

The main process in SYMGraph is that, firstly, it feeds the question and passage vector represented by BERT into a numerically-aware graph neural network. Then, a symbolic module produces compositional functions with predefined symbolic operators, which are executed similar to semantic parsing, to determine a final answer. This innovative paper combines the graph neural network and a symbolic module that generates functions, which are executed to produce answers. The main challenge when training SYMGraph is the lack of annotations, which was resolved with hard EM with thresholding. To tackle it, our training algorithm works by heuristically searching for programs that answer the question, and maximizing the likelihood of these programs with hard-EM-style parameter updates.

Related Work

Machine Reading Comprehension

Machine reading comprehension (MRC) is attracting more and more attention. Recently, some MRC tasks provided sufficient supervision of the answer spans have achieved a lot of notable results, such as SQuAD (Rajpurkar et al. 2016; **?**; Devlin et al. 2018; Seo et al. 2016; Xiong, Zhong, and Socher 2017). Recently, the community has paid attention to more challenging tasks, especially for semi-supervised MRC (Joshi et al. 2017), integrating reasoning about events (Kočiskỳ et al. 2018), and multi-hop reasoning (Dua et al. 2019; Yang et al. 2018). These unique datasets introduce new learning challenges since the supervision necessary to answer the question is not given. Nevertheless, there is still a lot of work not done yet for this particular learning challenge. Most work on machine reading comprehension concentrates on how to construct the model and randomly chooses a span from the given document (Joshi et al. 2017; Tay et al. 2018; Talmor and Berant 2019). From the perspective of human cognition, modeling the possible span choice is an essential process. Some other work maximizes the sum of the likelihood in their model to avoid the failure (Kadlec et al. 2016; Swayamdipta, Parikh, and Kwiatkowski 2017; Clarke et al. 2010; Lee, Chang, and Toutanova 2019), but it doesn't get a satisfactory result.

Discrete Reasoning Over Paragraphs

In order to improve arithmetic performance, we turn to diction and syntax in a particular word problem. Essentially, we are attempting to perform numerical reasoning alongside reading comprehension, as we usually need to evaluate a mathematical expression described in a passage to arrive at the correct answer. In some work (Kushman et al. 2014; Wang, Liu, and Shi 2017), the general approach is to generate equation templates based on the word problem, and map numbers into the templates. Kushman et al. use the training set to obtain a set of possible templates, which are selected and filled at inference time, while Wang, Liu, and Shi use a RNN to encode the word problem to a context vector and another RNN to decode the context vector to an equation template.

In recent years, a new dataset, DROP, introduced questions that were expected to present a harder challenge for reading comprehension models (Dua et al. 2019). At the beginning Dua et al. used QANET (Yu et al. 2018) as the encoder and added four different output layers onto it, including Passage Span, Question Span, Count and Arithmetic. Then, Min et al. extended a BERT-based extractive reading comprehension model to replace QANET with a lightweight extraction and composition layer. MTMSN (Hu et al. 2019) is the first to address the multi-span questions of DROP, which consists of dedicated categorical variable and the non-maximum suppression (NMS) algorithm to find the most probable set of non-overlapping spans. Following this, Efrat, Segal, and Shoham proposed a new approach for addressing multi-span questions based on sequence tagging. From a different perspective, Min et al. considered to derive some different mathematical equations by focusing the numbers in the reference text to calculate the answer. They predicted the most likely answer in the pre-computed span set with a discrete latent variable algorithm and then increased the probability of that answer. In the latest work, Ran et al. proposed a model named NumNet, especially for numerical reasoning, which utilizes a numerically-aware GNN to achieve numerical reasoning. They firstly extracted the number and then constructed the graph. However, it still divided the answer into four categories, which is not scalable to a multi-domain question-answering dataset because it relies on pre-defined answer decoders.

Passage	**Question & prediction**
Hoping to rebound from their loss to the Patriots, the Raiders stayed at home for a Week 16 duel with the Houston Texans. Oakland would get the early lead in the first quarter as quarterback **JaMarcus Russell completed a 20-yard touchdown pass** to rookie wide receiver Chaz Schilens. The Texans would respond with fullback Vonta Leach getting a 1-yard touchdown run, yet the Raiders would answer with kicker Sebastian Janikowski getting a 33-yard and a 30-yard field goal. Houston would tie the game in the second quarter with kicker Kris Brown getting a 53-yard and a 24-yard field goal. Oakland would take the lead in the third quarter with wide receiver **Johnnie Lee Higgins catching a 29-yard touchdown pass** from **Russell, followed up by an 80-yard punt** return for a touchdown…	**Question**: Who threw the longest pass?
	Predicted program: ARGMAX(("Russell", 20), ("Johnnie Lee Higgins", 29), ("Russell", 80)) **Result**: "Russell"
	Question: How many yards was the longest touchdown of the game?
	Predicted program: MAX(20, 1, 29, 80) **Result**: 80
	Question: How many yards longer was the 80-yard punt than the touchdown pass from Russell to Johnnie Lee Higgins?
	Predicted program: DIFF(80, 29) **Result**: 51
… the crown was officially pegged to **the mark** at a ratio of 1:10, even though the unofficial exchange rate was 1 to 6-7 and Germans immediately started buying Czech goods in large quantities…	**Question**: What is the currency of Germany called?
	Question: PASSAGE_SPAN(146, 148) **Result**: "the mark"

Figure 1: Some examples of operators, passages, question-functions and results.

SYMGraph

In this section, we will introduce the architecture of our model SYMGraph and describe the details of it for discrete

Operator	Arguments	Outputs	Description
SPAN_p SPAN_q	a0: the start index. a1: the end index.	a span.	Select a span from the passage or question.
VALUE	a0: index.	a number.	Select a number.
KV	a0: a span. a1: a number.	a key-value pair.	Select a key-value.
DIFF SUM	a0: index. a1: index.	a number.	Compute the difference sum of two numbers.
COUNT	a: spans.	a number.	Count the number of given spans.
MAX MIN	a: numbers.	a number.	Select the maximum. or minimum.
ARG	a: key-value	a span.	Select the argmax or argmin one

Table 1: The illustration of different operators.

reasoning over paragraphs. This is further illustrated in Figure 1.

Components

- Numerical Encoder: encodes question and passage into a vector, which represents a semantic meaning.
- Graph Neural Network: deciphers the relationship between the numbers in a paragraph.
- Symbolic Module: generate and execute the symbolic functions with pre-defined operators.

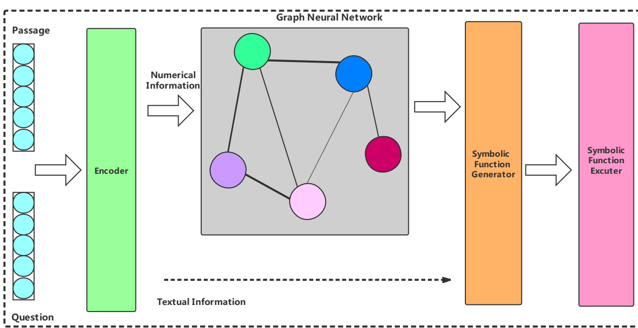

Figure 2: The structure of SYMGRAPH. It can be divided into three parts which are encoder, graph neural network and symbolic module.

Numerical Encoder

In this part, we will thoroughly discuss the comparison to related graph neural network, in particular, the differences between NumNet(Ran et al. 2019) and our work, as they combine a Bert-style encoder with a numerically aware graph neural network. Our model outputs a program after the graph neural network, rather than having several different output layers dependent on the question type.

We consider the numbers from the text, including question and passage, as nodes in the graph for the reasoning. The encoding module includes BERT part and GNN part. For NumNet, there major contribution is the unique reasoning module, thus their work can be considered as a combination of NAQANet and GNN. And our graph encoding part is the same as theirs.

Specifically, for the embedding representation of encoder, we feed the question and passage jointly into BERT. The question Q and passage P are first encoded as below, followed by the passage-aware question representation and the question-aware passage representation:

$$P = \mathbf{BERT} - \mathbf{Encoder}(P) \tag{1}$$

$$Q = \mathbf{BERT} - \mathbf{Encoder}(Q) \tag{2}$$

$$\hat{P} = \mathbf{Attention}(P, Q); \hat{Q} = \mathbf{Attention}(Q, P) \tag{3}$$

Graph Construction

Compared with NumNet, we don't utilize the graph to achieve supplementary reasoning, our aim is to extract numbers and enhance its representation. The intention is based on the fact that BERT cannot distinguish the meanings of different numbers clearly. We treat the numbers from the passage as nodes in the graph, which can be denoted as V_Q and V_P, respectively arise in question and passage. We symbolize the sets of nodes as $V = V_Q \cup V_P$, and $n(v)$ is the number of nodes V. In the beginning, we will initialize heterogeneous directed graph $G = (V; E)$. During the construction step, we point that the nodes and edges in the question and passage are V and E.

We have two nodes $v_i, v_j \in V$, for each node $v_i^P \in V_P$, it is defined as the corresponding column vector of M_P. The first vector is $v_i^P = M^P[I^P(v_i^P)]$, where $I^P(v_i^P)$ indicates the word index of v_i^P . In an identical way, the initial vector v_j^Q for a node $v_j^Q \in V_Q$ is fixed as the corresponding column vector of M_Q. We describe the node vector as $v_0 = v_i^P \cup v_j^Q$ and use a GNN to express graph G and use v to represent the node. Then, we calculate the weight of each node for further computation.

$$\alpha_i = sigmoid(W_v v[i] + b_i) \tag{4}$$

In equation, W_v is the weight of a node v_i, and b_v is a bias. We employ relation-specific transform matrices in the process of message propagation and thus in the following we define the propagation function, which aims at computing the forward-pass.

$$\hat{v}_i = \frac{1}{|N_i|}(\sum \alpha_j W^{r_{ji}} v[j]) \tag{5}$$

where for each node v_i, the representation can be indicated as $\hat{v}_i$, and the neighbors of v_i can be indicated as $N_i = \{j|(v_j, v_i) \in E\}$. For each edge e^i_j, r^i_j is the relation between them. And also, $W^{r_{ji}}$ is relation-specific transform matrices.

Additionally, each node of the graph should also embody its own information as well, not only the neighbors' information.

$$v = \boldsymbol{ReLU}(W_f v_i + \hat{v}_i + b_f) \tag{6}$$

Finally, we indicate the above process as a single function to obtain a numerical relationship. And in this way, the output embedding of graph encoder will then concatenate to pre-trained model's embedding.

$$v = \mathrm{GCN}(G, v) \tag{7}$$

Symbolic Module

Although NumNet(Ran et al. 2019) also combines a BERT-style encoder with a numerically-aware graph neural network, there are huge differences in our model. In particular, the differences between NumNet and this work are more thoroughly discussed in this part, which can be summarized that our model generates a program and executes it directly, instead of using expression templates

And most importantly, we design ten operators for this task of discrete reasoning over paragraphs: DIFF, SUM, COUNT, ARG, MAX, MIN, PASSAGE-SPAN, QUESTION-SPAN, VALUE, KEY-VALUE. The arguments and description of these operators are shown in Table 1. Moreover, these operators can be combined to obtain the target function.

Since it may be hard to understand what some of the operators are by its name, we give some examples (Figure 2) and each operator has one or two arguments. PASSAGE-SPAN and QUESTION-SPAN select a span from the text, whose arguments are the start index and the end index. For COUNT (count the number of given spans), MAX and MIN, these three aims at some spans or numbers. The arguments of KEY-VALUE are a span and a number, which select a key (span) value (number) pair from the passage. The argument of VALUE is a index, which select a number from the text. Finally, ARGMAX and ARGMIN's arguments are some key-value pairs, and which select the key (span) with the highest / lowest value.

There are some following problems, we do not have correct function and this incomplete data with a lack of the target program cannot be directly used in training. What is more, there must be only one function corresponding to the question, but actually we may produce a wrong function which leads a correct answer as well. Firstly, we try to find programs for questions answerable by span selection or arithmetic operations via an exhaustive search. But actually for counting or sorting, the space becomes too large for an exhaustive search and usually includes more than one span as an argument. Thus, we augment the span selection questions by replacing the interrogatives For counting problem and extract the key-value pairs with entity recognition bash for sorting problem.

Hard EM

Another problem encountered in training the program was when a wrong program may predict a right answer. Hard EM algorithm selects a program with the highest probability between these programs, although these programs all return the correct answer. And then, the model maximizes the likelihood of the selected program.

However, there exists the third question that the annotated answer itself may is wrong. Since the hard EM algorithm is applied, though the probabilities for all these programs may be tiny, the model can still select the appropriate program for the training step. In the future, we can adopt RL-based approaches such as MAPO solve it.

Hard EM Algorithm The input is question-answer pairs (x_i, y_i), we apply Data Augmentation for each (x_i, y_i) to produce a program set. Then repeat: (a) get a new threshold*=decay and D is a empty set; (b) for each (x_i, y_i): we can pick a $program = argmax_k p_\theta(program|x_i)$ form the program set and if $p_\theta(program|x_i) > threshold$ and program set=1 then $D = D + (X_i, program)$; (c) Update θ by maximizing $\sum_D logp_\theta(program|x)$ until converge or early stop.

Experiment

Dataset and Evaluation Metrics

Our proposed model is evaluated on the DROP dataset (Dua et al. 2019), which is a public numerical MRC dataset. DROP dataset is constructed by crowd-sourcing, and the authors ask the annotators to generate question-answer pairs according to some given Wikipedia passages. Unlike the other datasets based on Wikipedia, DROP emphasize the numerical reasoning, such as addition, counting, and sorting over numbers, called as discrete reasoning. In details, DROP is divided into training set, which contains 77,409 samples, development set, which contains 9,536 samples, and 9,622 testing samples. These specifications can be found in Table 4. For this study and experiment, we adopt two metrics: (1) Exact Match (EM) and (2) numerically-focused F1 scores to evaluate our model following (Dua et al. 2019). If the predicted answer is mismatched for those questions with the numeric correct answer, F1 is set to 0, otherwise it will be 1.

Baselines

In order to comparison, we select several public models as baselines, which are described as below. It's worth noting that we only compared with the single model approaches, and we don't consider some enhanced version, such as using a pretrained RoBERTa model.

- Syn Dep (Dua et al. 2019), the neural semantic parsing model (KDG) (Krishnamurthy, Dasigi, and

Model	EM (Dev)	F1 (Dev)	EM (Test)	F1 (Test)
Heuristic Baseline (Dua et al. 2019)	4.28	8.07	4.18	8.59
Syn Dep (Dua et al. 2019)	9.38	11.64	8.51	10.84
OpenIE (Dua et al. 2019)	8.80	11.31	8.53	10.77
Semantic Role Labeling (Carreras and Màrquez 2004)	9.28	11.72	8.98	11.45
BiDAF (Seo et al. 2016)	26.06	28.85	24.75	27.49
QANet+ELMo (Yu et al. 2018)	27.71	30.33	27.08	29.67
BERT BASE (Devlin et al. 2018)	30.10	33.36	29.45	32.70
NAQANet (Dua et al. 2019)	46.20	49.24	44.07	47.01
NABERT+	62.59	66.46	61.60	65.12
MTMSN LARGE (Efrat, Segal, and Shoham 2019)	76.54	80.76	75.88	79.99
NumNet (Ran et al. 2019)	64.92	68.31	64.56	67.97
SYMGraph (Ours)	**81.24**	**83.09**	**79.06**	**82.96**

Table 2: Comparing test and development set results of models from the official DROP leaderboard.

Model	**Multi-Span (4.8%)**		**Single-Span (31.7%)**		**Number (61.9%)**		**Date (1.6%)**	
	EM	F1	EM	F1	EM	F1	EM	F1
NAQANet	0.00	27.33	58.24	64.82	44.97	45.09	32.03	39.61
NABERT	6.53	27.48	65.56	70.62	54.27	54.29	37.58	46.38
MTMSN	25.17	62.86	77.58	82.88	80.93	81.15	55.71	69.07
SYMGraph (test set)	52.18	78.32	75.98	80.68	83.25	84.87	59.12	68.18

Table 3: Performance of different models on DROP's test/development set in terms of Exact Match (EM) and F1.

Statistic	**Train**	**Dev**
Num passages	5565	582
Num questions	77409	9536
Avg questions / passage	13.91	16.38
Avg answer / question	1.00	3.35
Avg passage len (tokens)	278.39	256.10
Avg question len (tokens)	12.89	13.39
Passage vocab size (tokens)	20986	11102
Question vocab size (tokens)	14051	5794

Table 4: DROP dataset specs.

Gardner 2017) with Stanford dependencies based sentence representations;

- OpenIE (Dua et al. 2019), KDG with open information extraction based sentence repre- sentations;
- SRL (Dua et al. 2019), KDG with semantic role labeling based sentence representations;

general reading comprehension models:

- BiDAF (Seo et al. 2016), which utilizes Bi-Directional Attention Flow as a multi-stage hierarchical process;
- QANet (Yu et al. 2018), which utilizes convolutions and self-attentions as the building blocks of encoders to represent the question and passage;
- BERT (Devlin et al. 2018), Bidirectional Encoder Representations from Transformers (BERT) is a technique for NLP (Natural Language Processing) pre-training developed by Google, which gets state-of-the-art performance on various reading comprehension datasets;

and numerical reading comprehension models:

- NAQANet (Dua et al. 2019), a numerical version of QANet model.
- NAQANet+, an enhanced version of NAQANet implemented by ourselves, which further considers real numbers (e.g. "2.5"), richer arithmetic expression, data augmentation, and more. The enhancements are also used in our NumNet model and the details are given in the Appendix.
- NumNet (Ran et al. 2019), considers numerical comparing information among numbers when answering numerical questions. We don't compare with some enhanced versions for the fairness, because our intention doesn't include the tricks of MRC task.

Result

Two metrics, namely Exact Match (EM) and F1 score, are utilized to evaluate models. We use the official script to compute these scores. Since the test set is hidden, we only submit the best single model to obtain test results. Table 2 shows the performance of our model and other competitive approaches on the development and test sets. SYMGraph outperforms all existing approaches by a large margin, and creates new state-of-the-art results by achieving an EM score of 79.06 and a F1 score of 82.96 on the test set. Since our best model utilizes BERT LARGE as encoder, we therefore compare our model with MTMSN LARGE and NABERT LARGE baseline. As we can see (Table 3), our model obtains 3.18/2.97 absolute gain of EM/F1 over MTMSN LARGE and 34.99/35.95 absolute gain of EM/F1 over NABERT LARGE, demonstrating the effectiveness of our approach. However, as the human achieves 95.98 F1 on the test set, our results suggest that there is still room for improvement.

Analysis

There are 21,800 questions chosen from the original DROP dataset (Dua et al. 2019) in our experiment. The questions we used are suitable for our symbolic module; thus, it can be a useful dataset to prove our proposed model's effectiveness. From the results, we can find that our model outperforms the existing state-of-the-art models on DROP. What's more, our model first generates the function; thus, it could be interpretable and has more advantages than black-box models.

Conclusion and Future Work

In this paper, we explore the problem of discrete reasoning over paragraphs, including arithmetic, counting, and sorting, where complex questions could be correctly answered. To tackle this, we propose SYMGraph, a semantic parser that operates over passages and coupled with a numerically-aware graph. Our model handle the last restriction of existing graph model, such as NumNet, having output layers restricted to the question type. The key insight is to let the semantic parser point to locations in the text that can be used in further symbolic functions in a Hard-EM training. Then, we describe the SYMGraph and show that it can effectively answer questions better than baseline models on DROP datasets. Specifically, SYMGraph combines symbolic methods and neural networks, such as an advanced language model (BERT (Devlin et al. 2018)) and represents learning (graph neural network), enabling them to achieve numerical reasoning efficiently. Our work better defines the issues with the neural symbolic approach and sets the stage for future work on this problem. We plan to generalize this model to more complex and compositional machine reading comprehension data-sets with better strategies of the neural symbolic approach.

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